Generalized Wavelet Based Ratio Operators and Polarimetric SAR

Wavelet Based Polarimetric SAR Image Time Series Decompositions

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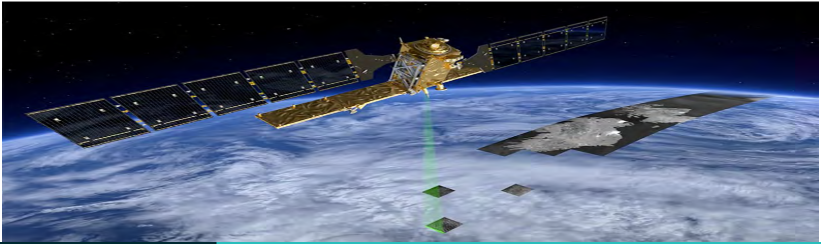


Data Characteristics / Issues

- “High” resolution in time (increasingly small revisit time)

  - *[RADARSAT-2]*: repeat cycle in 24 days.
  - *[Sentinel-1A]*: repeat cycle in 12 days.

- “High” spatial resolution (resolution of 1 to 3m).

  - *[RADARSAT-2]*: 5m $\times$ 5m (fine quad-PolSAR).
  - *[Sentinel-1A]*: 3.5m $\times$ 20m (dual-PolSAR IW SLC).

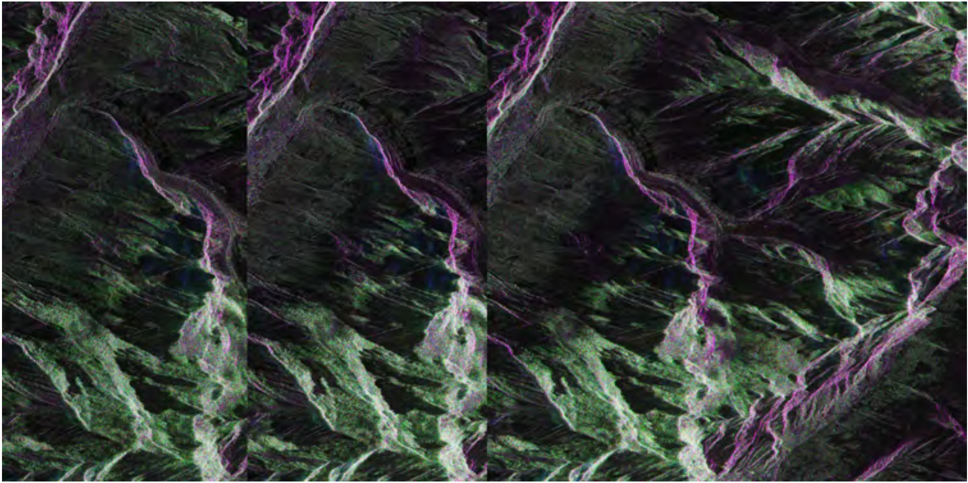
- Image Time Series (ITS) involving polarimetry information (PolSAR-ITS)

  - *[Test Site]*: French Alps glaciers, scene with size 10/20km $\times$ 10km.
  - *[RADARSAT-2, Sentinel-1A]*: (test site) multivariate images $\approx 5 \times 10^7$ pix.

- **Issue:** statistical processing / change detection from stochastic big data.
- **Constraint:** seeking performance, robustness (to a wide range of changes) and low computational load.

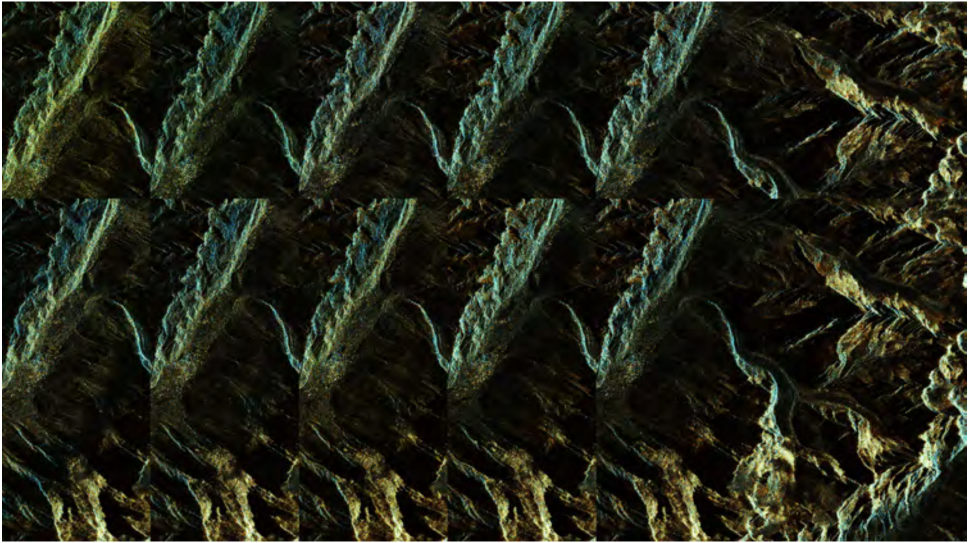
RADARSAT 2 / fine quad polarization

From 2009-03-18 to 2009-05-05



Sentinel-1A / dual VV and VH polarizations

From 2014-11-15 to 2015-03-03



Data Analysis

Content

- High Performance Analysis (HPA)
- High Performance Computing (HPC)
- Multi-Date Change Analysis
- Application to RADARSAT-2 / Sentinel-1A Image Time Series

Decomposition (polarimetry)

- discriminant information analysis.

Transform (parsimony)

$\Longrightarrow$ geometric features enhancement,

$\Longrightarrow$ separation: smooth (approximations) versus singularities (changes, details).

Transform (Arithmetic)

Sparse wavelet details '+' *Stationary noise*

Multiplicative interaction model

Function f is sparse in $\mathcal{W} = [\mathcal{W}^A, \mathcal{W}^D]$ domain, where $\mathcal{W}$ is a linear transform, but we observe:

$$f(k) + X(k)$$

- f is a deterministic and regular function;
- $(X(k))_k$ are stationary zero-mean random variables;
- Sequence X is independent with f;
- $\mathcal{W}^A$ is a generalized approximation (mean, trend) operator;
- $\mathcal{W}^D$ is a generalized differencing (details) operator.

Linearity and sparsity — Parsimonious framework

$$\mathcal{W}[f + X] \quad = \quad [\mathcal{W}^A f] + [\mathcal{W}^A X] \quad + \quad [\mathcal{W}^D f] + [\mathcal{W}^D X]$$

- $[\mathcal{W}^D f] \mapsto$ Mainly singularities of f ?
- $[\mathcal{W}^D X] \mapsto$ Mainly stationary and decorrelated noise ?

Transform (Arithmetic) SWT

“FSAR” - Original PolSAR Image

“FSAR” - PolSAR SWT / $\mathcal{W}^D$ / $J = 1$

Transform (Arithmetic) SWT

"FSAR" - Original PolSAR Image

"FSAR" - PolSAR SWT / $\mathcal{W}^D$ / $J = 3$

Transform (Geometric)

Sparse wavelet details '+' *Stationary noise*

Multiplicative interaction model

Function f is sparse in $\mathcal{W}$ domain, where $\mathcal{W}$ is a linear transform, but we observe:

$$f(k)X(k) = \begin{cases} f(k) + f(k)[X(k) - 1] & (\star) \\ e^{\log f(k) + \log X(k)} & (\star\star) \end{cases}$$

- f and X are strictly positive function and random sequence,
- $(X(k))_k$ are stationary random variables and X is independent with f.

Standard (additive) wavelet transform framework — Model $(\star)$

$$\mathcal{W}fX = [\mathcal{W}f] \quad \oplus \quad \{\mathcal{W}f(X-1)\} \mapsto \text{Details ?}$$

Geometric (multiplicative) wavelet transform framework — Model $(\star\star)$

Wavelet in a multiplicative algebra [binary operations $(\oplus, \otimes) = (\times, \wedge)$] or *geometric wavelet*, $\mathcal{W}$, involves geometric approximations and differencing, with:

$$\mathcal{W}fX = [\mathcal{W}f] \quad \oplus \quad \{[\mathcal{W}X]\} \mapsto \text{Details ?}$$

Transform (Geometric)

[Atto, Trouvé, Nicolas, Lê, 2016]

Geometric convolution

Let $\mathbf{h} = (\mathbf{h}[\ell])_{\ell \in \mathbb{Z}}$ denotes the impulse response of a digital filter. We define the geometric convolution of $\mathbf{x}$ and $\mathbf{h}$ on the vectorial space $(\mathbb{R}^+, \times, \wedge)$ as:

$$\mathbf{y}[k] = \mathbf{x} \circledast \mathbf{h}[k] \triangleq \prod_{\ell \in \mathbb{Z}} (\mathbf{x}[\ell])^{\mathbf{h}[k-\ell]} = \prod_{\ell \in \mathbb{Z}} (\mathbf{x}[k-\ell])^{\mathbf{h}[\ell]} \triangleq \mathbf{h} \circledast \mathbf{x}[k], \qquad (1)$$

Transform (Geometric)

[Atto, Trouvé, Nicolas, Lê, 2016]

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Geometric wavelet decomposition

Define the *geometric wavelet decomposition* of $\mathbf{x}$ by:

$$\mathbf{c}_{1,0}[k] = \mathbf{x} \circledast \overline{\mathbf{h}_0}[2k], \qquad \mathbf{c}_{1,1}[k] = \mathbf{x} \circledast \overline{\mathbf{h}_1}[2k], \qquad (2)$$

and, recursively, for $\epsilon \in \{0,1\}$ (approximations when $\epsilon = 0$ and details when $\epsilon = 1$):

$$\mathbf{c}_{j+1,2n+\epsilon}[k] = \mathbf{c}_{j,n} \circledast \overline{\mathbf{h}_\epsilon}[2k]. \qquad (3)$$

Transform (Geometric)

[Atto, Trouvé, Nicolas, Lê, 2016]

Geometric convolution

Let $\mathbf{h} = (\mathbf{h}[\ell])_{\ell \in \mathbb{Z}}$ denotes the impulse response of a digital filter. We define the geometric convolution of $\mathbf{x}$ and $\mathbf{h}$ on the vectorial space $(\mathbb{R}^+, \times, \wedge)$ as:

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$$\mathbf{c}_{j+1,2n+\epsilon}[k] = \mathbf{c}_{j,n} \circledast \overline{\mathbf{h}_\epsilon}[2k]. \quad (3)$$

Geometric wavelet reconstruction

$$\mathbf{c}_{j,n}[k] = \left(\check{\mathbf{c}}_{j+1,2n} \circledast \mathbf{h}_0[k]\right) \times \left(\check{\mathbf{c}}_{j+1,2n+1} \circledast \mathbf{h}_1[k]\right), \quad (4)$$

$$\check{\mathbf{u}}[2k+\epsilon] = \begin{cases} \mathbf{u}[k] & \text{if} \quad \epsilon = 0, \\ 1 & \text{if} \quad \epsilon = 1. \end{cases} \quad (5)$$

Transform (Geometric) — Level 1 details

Geometric differencing $\Rightarrow$ *ratioing* of $(\mathbb{R}^+, \times)$ elements

Examples of level-1 wavelet based ratioing

- Case of the Haar wavelet:

$$\frac{y_k}{y_{k-1}} \tag{6}$$

Remark: *arithmetic differencing* involves $y_k - y_{k-1}$.

- Case of a biorthogonal wavelet with 2 vanishing moments:

$$\frac{y_{k-1}^{0.35} y_{k+1}^{0.35}}{y_k^{0.70}} \tag{7}$$

- Case of a box spline wavelet with 2 vanishing moments:

$$\frac{y_{k-1}^{0.6875} y_{k-2}^{0.21875} y_{k-3}^{0.03125}}{y_k^{0.6875} y_{k+1}^{0.21875} y_{k+2}^{0.03125}} \tag{8}$$

Transform SWT *versus* Geometric SWT (GSWT)

“FSAR” - Original PolSAR Image

“FSAR” - PolSAR SWT / $\mathcal{W}^D$ / $J = 3$

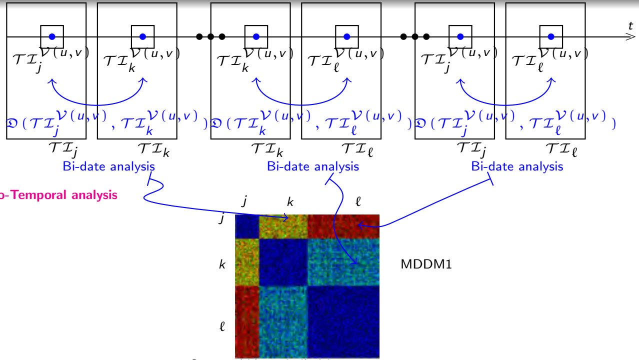
Transform SWT *versus* Geometric SWT (GSWT)

"FSAR" - Original PolSAR Image

"FSAR" - PolSAR GSWT / $\mathcal{W}^D$ / $J = 3$

Analysis / Image Time Series [Atto, Trouvé, Berthoumieu, Mercier], [Lê, Atto, Trouvé]

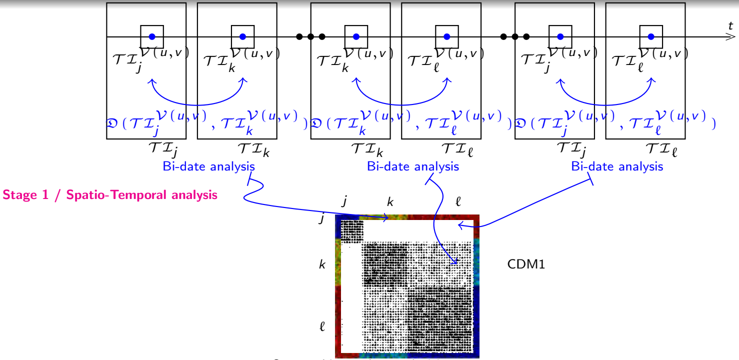
Stage 1 / Spatio-Temporal analysis



Stage 1 bidate similarities

Analysis / Image Time Series

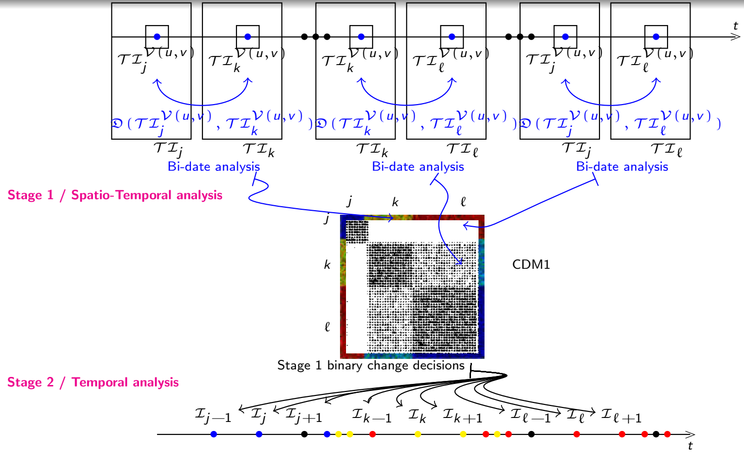
[Atto, Trouvé, Berthoumieu, Mercier], [Lê, Atto, Trouvé]



Stage 1 binary change decisions

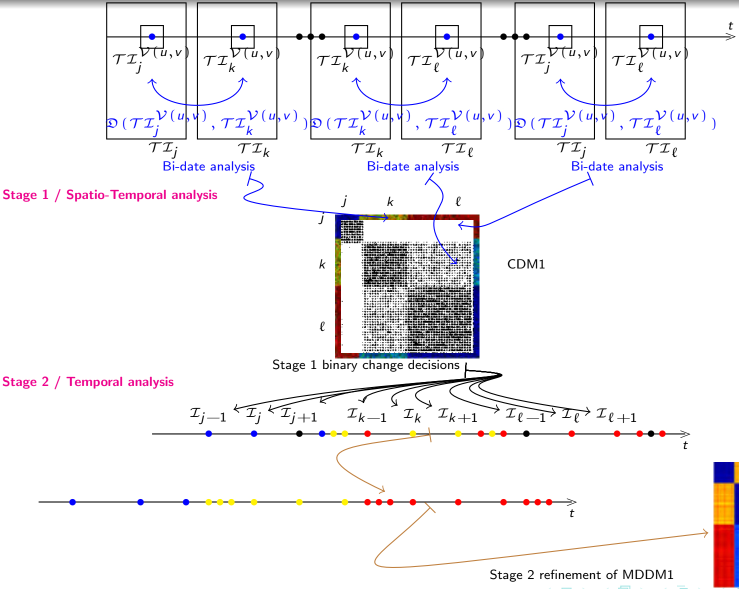
Analysis / Image Time Series

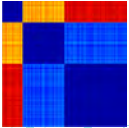
[Atto, Trouvé, Berthoumieu, Mercier], [Lê, Atto, Trouvé]



Analysis / Image Time Series

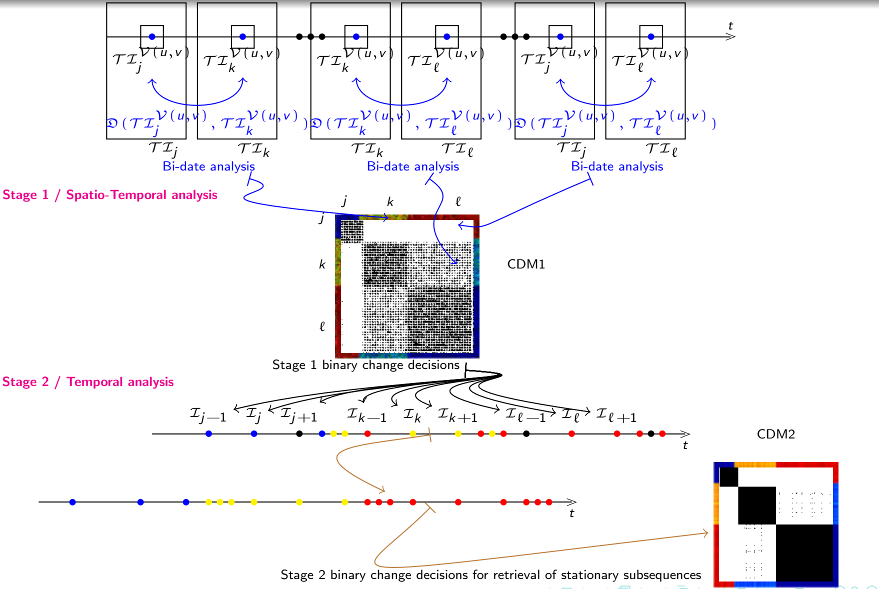
[Atto, Trouvé, Berthoumieu, Mercier], [Lê, Atto, Trouvé]





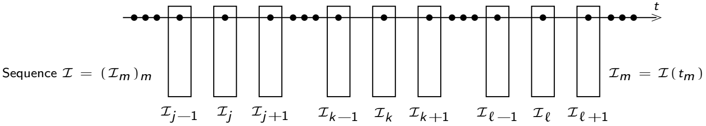
Analysis / Image Time Series

[Atto, Trouvé, Berthoumieu, Mercier], [Lê, Atto, Trouvé]



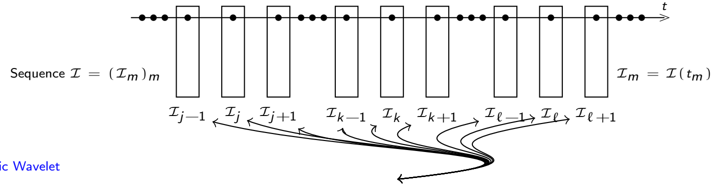
Analysis / Image Time Series

[Atto, Trouvé, Nicolas, Lê]



Analysis / Image Time Series

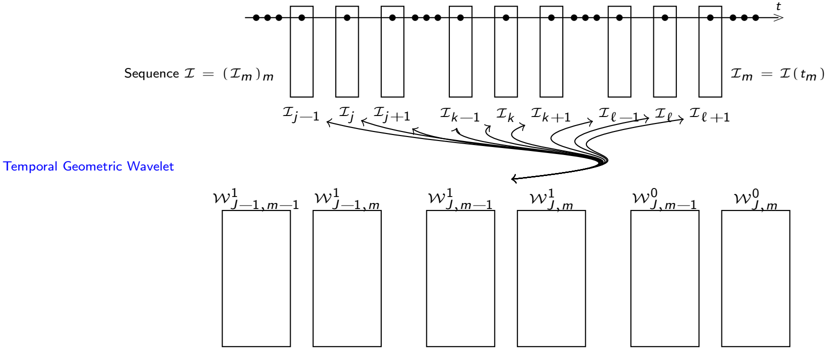
[Atto, Trouvé, Nicolas, Lê]



Temporal Geometric Wavelet

Analysis / Image Time Series

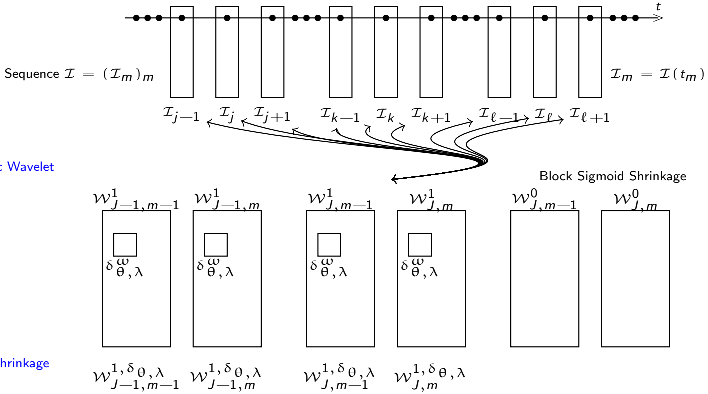
[Atto, Trouvé, Nicolas, Lê]



Analysis / Image Time Series

[Atto, Trouvé, Nicolas, Lê]

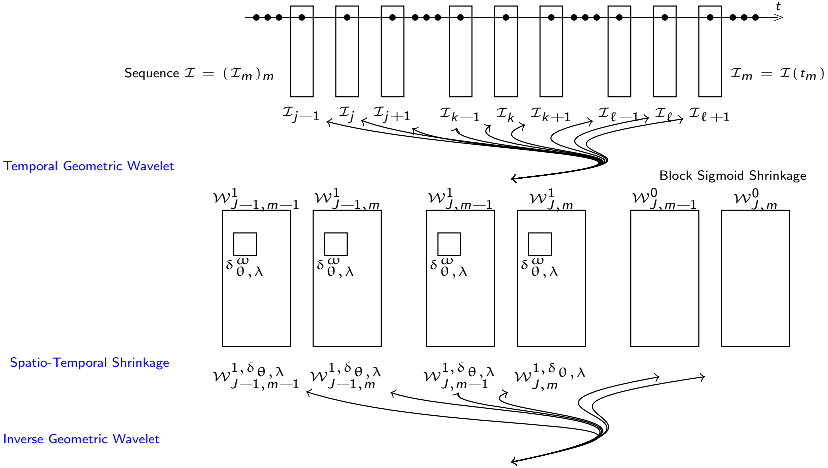
Temporal Geometric Wavelet



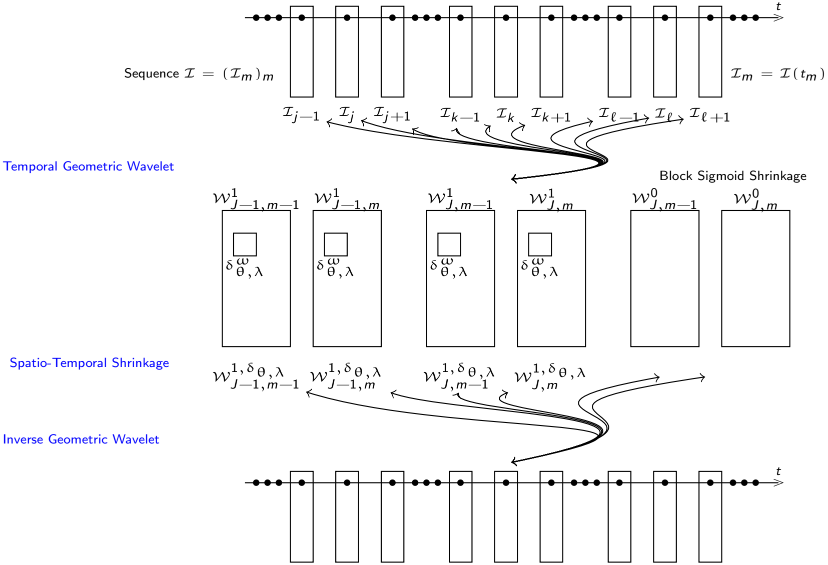
Spatio-Temporal Shrinkage

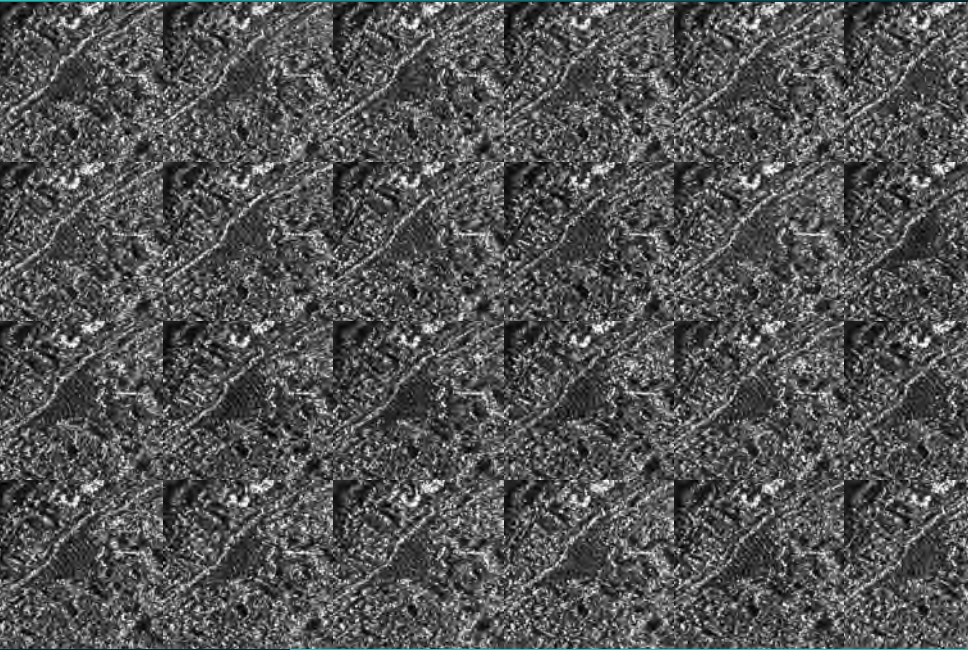
Analysis / Image Time Series

[Atto, Trouvé, Nicolas, Lê]



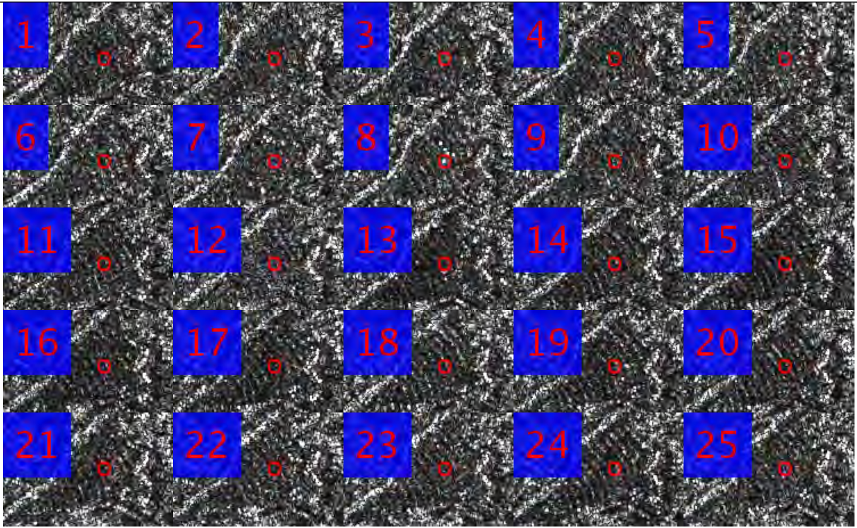
Analysis / Image Time Series [Atto, Trouvé, Nicolas, Lê]





Transform

Focus on temporal variable



PolSAR features

Pixel information

Vector features

[Cases: "Lexicographic" and "Pauli"]

$$\underline{\mathbf{k}}_{3L} = \frac{1}{\sqrt{2}} \begin{pmatrix} S_{11}\sqrt{2} \\ S_{12}+S_{21} \\ S_{22}\sqrt{2} \end{pmatrix} \quad \underline{\mathbf{k}}_{3P} = \frac{1}{\sqrt{2}} \begin{pmatrix} S_{11}+S_{22} \\ S_{11}-S_{22} \\ S_{12}+S_{21} \end{pmatrix}$$

Scattering matrix: $\mathcal{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$

Covariance matrix: $\mathcal{C} = \underline{\mathbf{k}}_{3L}\overline{\underline{\mathbf{k}}_{3L}^*}$

Coherency matrix: $\mathcal{T} = \underline{\mathbf{k}}_{3P}\underline{\mathbf{k}}_{3P}^*$

Kennaugh matrix: $\mathcal{K} = f(\mathcal{T})$

Huynen params: $\mathbf{A}_0, \mathbf{B}_0, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$

Matrix features

[Cases: Covariance, Coherency and Kennaugh]

$$\mathcal{C} = \begin{pmatrix} |S_{11}|^2 & 2^{-\frac{1}{2}} S_{11}(S_{12}+S_{21})^* & S_{11}S_{22}^* \\ 2^{-\frac{1}{2}}(S_{12}+S_{21})S_{11}^* & 2^{-1}|S_{12}+S_{21}|^2 & 2^{-\frac{1}{2}}(S_{12}+S_{21})S_{22}^* \\ S_{22}S_{11}^* & 2^{-\frac{1}{2}} S_{22}(S_{12}+S_{21})^* & |S_{22}|^2 \end{pmatrix}$$

$$\mathcal{T} = \begin{pmatrix} 2\mathbf{A}_0 & \mathbf{C}-i\mathbf{D} & \mathbf{H}+i\mathbf{G} \\ \mathbf{C}+i\mathbf{D} & \mathbf{B}_0+\mathbf{B} & \mathbf{E}+i\mathbf{F} \\ \mathbf{H}-i\mathbf{G} & \mathbf{E}-i\mathbf{F} & \mathbf{B}_0-\mathbf{B} \end{pmatrix}$$

$$\mathcal{K} = \begin{pmatrix} \mathbf{A}_0+\mathbf{B}_0 & \mathbf{C} & \mathbf{H} & \mathbf{F} \\ \mathbf{C} & \mathbf{A}_0+\mathbf{B} & \mathbf{E} & \mathbf{G} \\ \mathbf{H} & \mathbf{E} & \mathbf{A}_0-\mathbf{B} & \mathbf{D} \\ \mathbf{F} & \mathbf{G} & \mathbf{D} & -\mathbf{A}_0+\mathbf{B}_0 \end{pmatrix}$$

PolSAR features

Parsimony and changes

PolSAR image time series & wavelets

- Image time series $\mathcal{I} = \{\mathcal{I}_k\}_{k=1,\ldots,M}$,
  - sequence with M images of the same scene (co-registration),
  - acquisition $\mathcal{I}_k = \mathcal{I}[t_k]$, where k/t_k refer to the acquisition date.
- Pixel $\mathcal{I}_k^{(u,v)}(x,y)$ with spatial location (x,y), sampling date k and polarimetric information (u,v) in terms of covariance, coherency and Kennaugh matrices:

$$\mathcal{I}_k^{\mathcal{C}}(x,y) = \mathcal{C}_k(x,y); \qquad \mathcal{I}_k^{\mathcal{T}}(x,y) = \mathcal{T}_k(x,y); \qquad \mathcal{I}_k^{\mathcal{K}}(x,y) = \mathcal{K}_k(x,y).$$

- Geometric transform $\mathcal{W}$ applies on temporal variable k, with

$$\mathcal{W} = [\mathcal{W}^A \quad \mathcal{W}^D] = \left[\mathcal{W}_J^A \quad \left(\mathcal{W}_j^D\right)_{1 \leqslant j \leqslant J}\right]$$

  detail $\mathcal{W}_j^D \mathcal{I} = \mathrm{d}_{\mathcal{W}_j} \mathcal{I}$ is a sequence of geometric change-images (scale j generalized log-ratio operator).

- Regularization of $\left(\mathcal{W}_j^D \mathcal{I}\right)_k$ applies on spatial and PolSAR variables (u, v, x, y).
- Level-j Wavelet Total Variation (WTV): $\sum_k \left|\left[\mathcal{W}_j^D \mathcal{I}\right]_k\right|$.

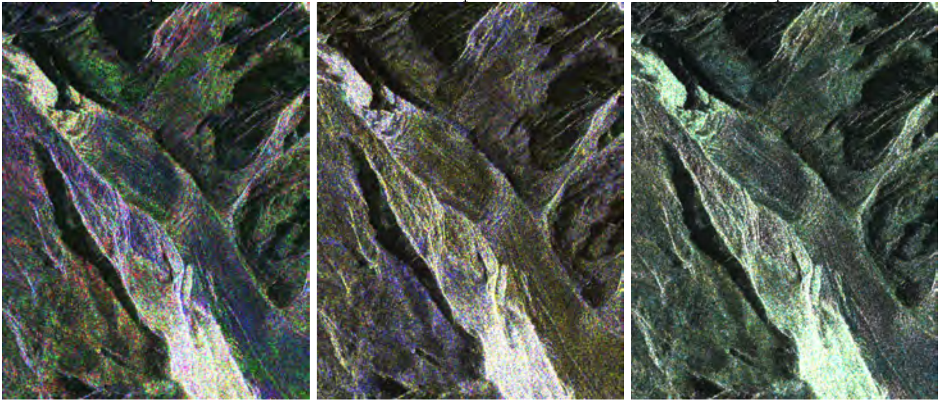
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg$\times$Az

$t_1 = 2009 - 02 - 22$

$\mathcal{I}_{t_1}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_1}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_1}^{\mathcal{K}\mathtt{diag}}$



Radarsat-2 / Full PolSAR

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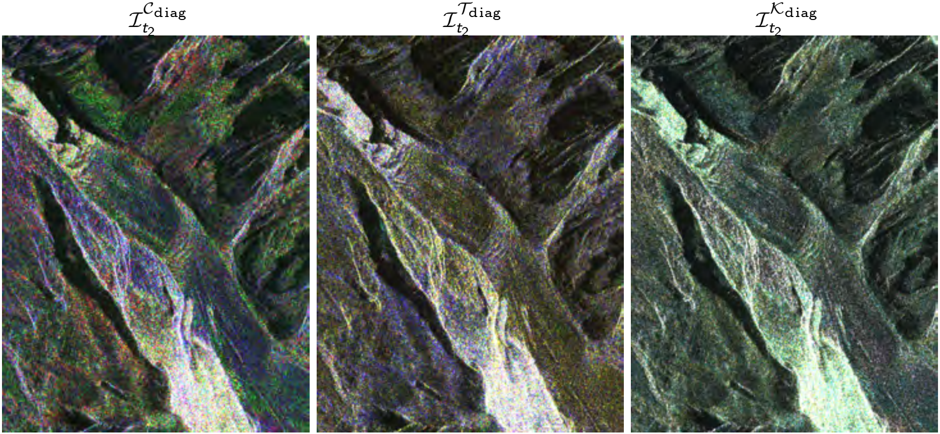
Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / 9m × 7.6m Rg×Az

$t_2 = 2009 - 03 - 18$



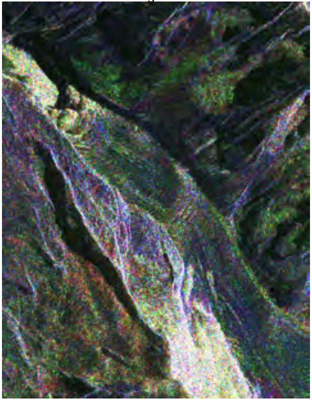
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

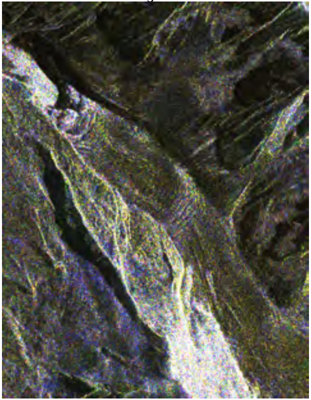
Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

$t_3 = 2009 - 04 - 11$

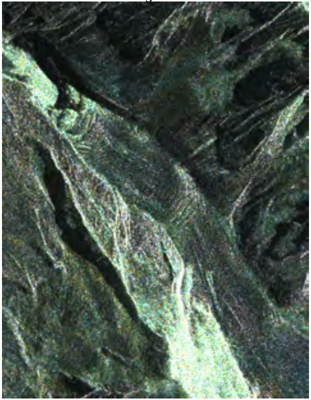
$\mathcal{I}_{t_3}^{\mathcal{C}_{\text{diag}}}$



$\mathcal{I}_{t_3}^{\mathcal{T}_{\text{diag}}}$



$\mathcal{I}_{t_3}^{\mathcal{K}_{\text{diag}}}$



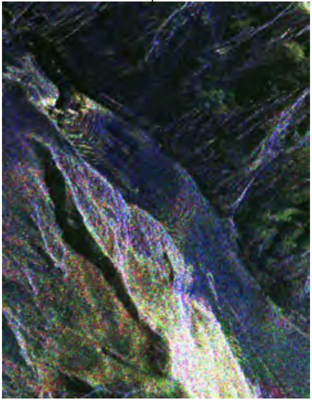
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

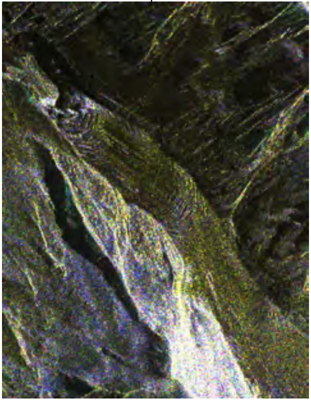
Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

$$t_4 = 2009 - 05 - 05$$

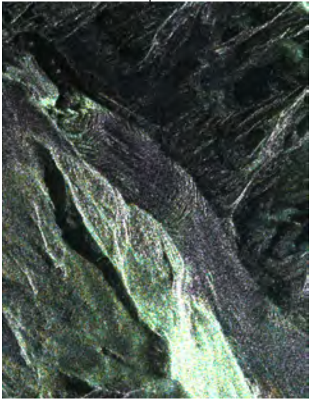
$\mathcal{I}_{t_4}^{\mathcal{C}\texttt{diag}}$



$\mathcal{I}_{t_4}^{\mathcal{T}\texttt{diag}}$



$\mathcal{I}_{t_4}^{\mathcal{K}\texttt{diag}}$



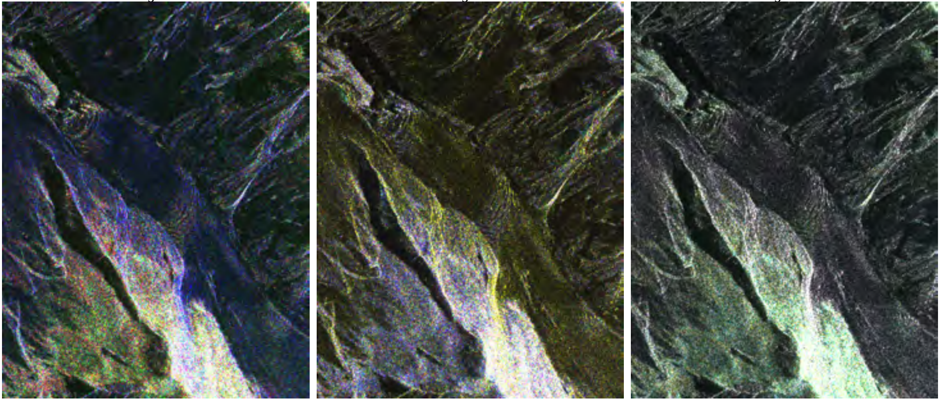
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

$t_5 = 2009 - 04 - 11$

$\mathcal{I}_{t_5}^{\mathcal{C}\text{diag}}$ $\mathcal{I}_{t_5}^{\mathcal{T}\text{diag}}$ $\mathcal{I}_{t_5}^{\mathcal{K}\text{diag}}$



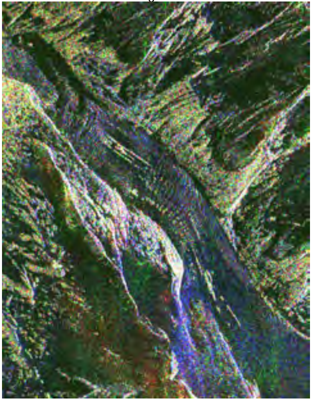
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

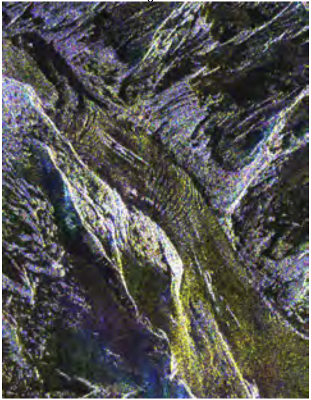
Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg$\times$Az

$$t_6 = 2009 - 03 - 18$$

$\mathcal{I}_{t_6}^{\mathcal{C}\,\mathtt{diag}}$



$\mathcal{I}_{t_6}^{\mathcal{T}\,\mathtt{diag}}$



$\mathcal{I}_{t_6}^{\mathcal{K}\,\mathtt{diag}}$

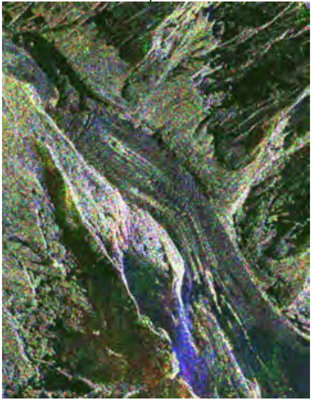
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

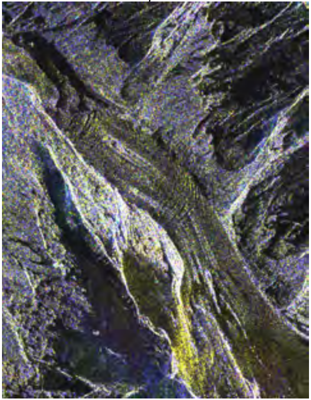
Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / 9m × 7.6m Rg×Az

$t_7 = 2009 - 02 - 22$

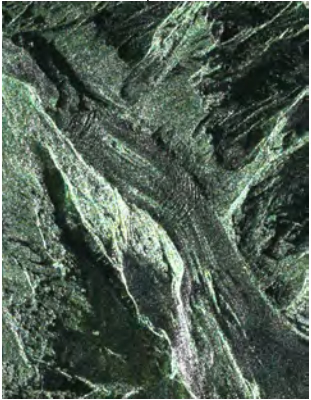
$\mathcal{I}_{t_7}^{\mathcal{C}\mathtt{diag}}$



$\mathcal{I}_{t_7}^{\mathcal{T}\mathtt{diag}}$



$\mathcal{I}_{t_7}^{\mathcal{K}\mathtt{diag}}$



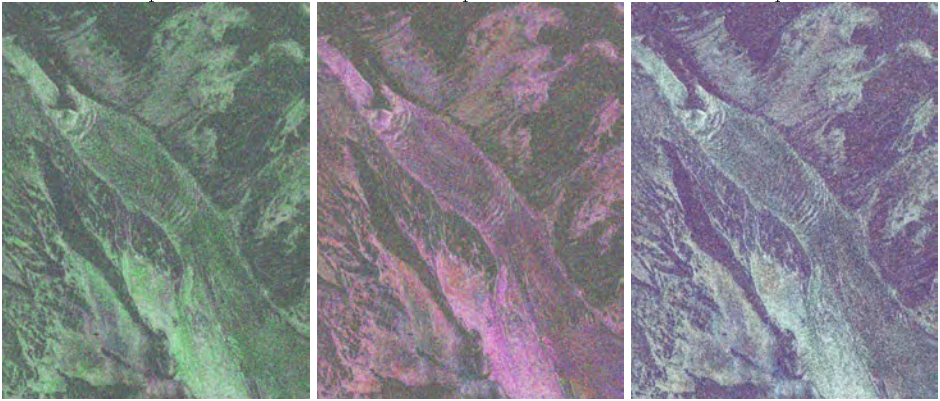
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

Wavelet Total Variation (WTV)

$\mathcal{I}_{t_1}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_1}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_1}^{\mathcal{K}\mathtt{diag}}$



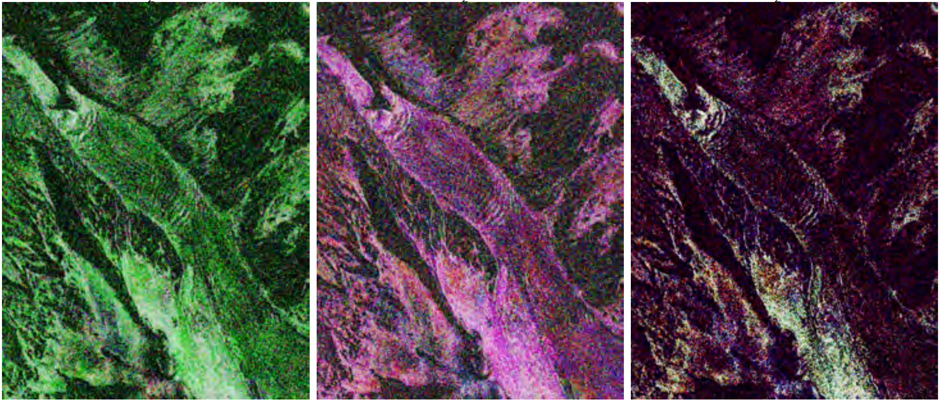
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

WTV PolSAR sigmoid shrinkage

$\mathcal{I}_{t_2}^{\mathcal{C}\,\mathtt{diag}}$ $\mathcal{I}_{t_2}^{\mathcal{T}\,\mathtt{diag}}$ $\mathcal{I}_{t_2}^{\mathcal{K}\,\mathtt{diag}}$



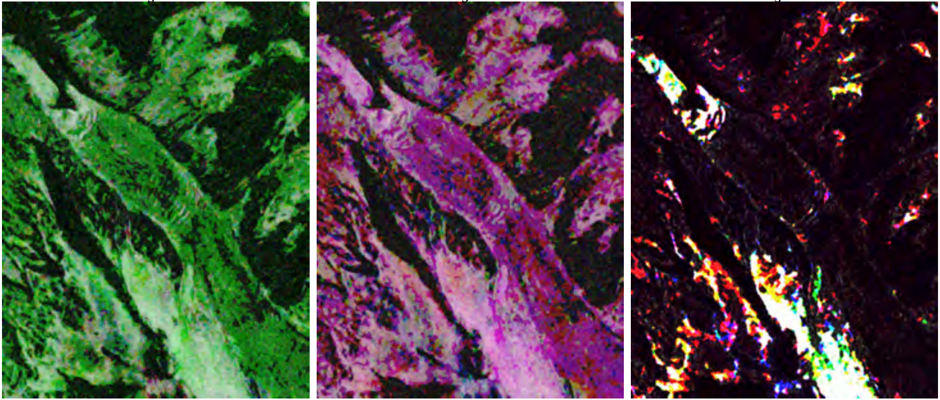
Radarsat-2 / Full PolSAR

Covariance, Coherency, Kennaugh

Fine Quad Polarization / SLC / Descending / 32° incidence / C-Band / $9m \times 7.6m$ Rg×Az

WTV spatial recursive $\ell_1 - \ell_2$ regularization

$\mathcal{I}_{t_3}^{\mathcal{C}_{\mathrm{diag}}}$ $\mathcal{I}_{t_3}^{\mathcal{T}_{\mathrm{diag}}}$ $\mathcal{I}_{t_3}^{\mathcal{K}_{\mathrm{diag}}}$



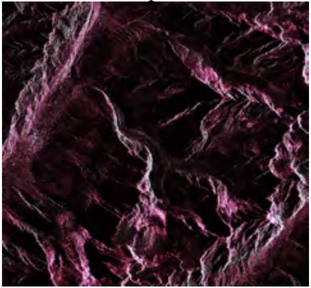
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

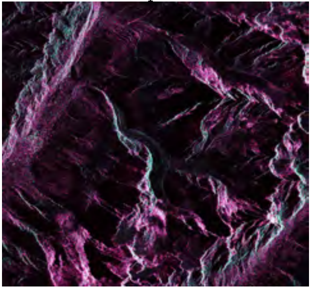
Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$t_1 = 2014 - 11 - 15$

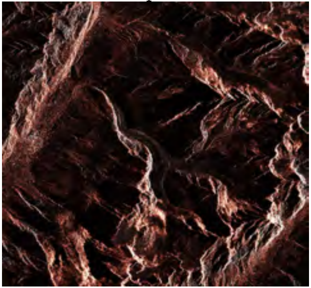
$\mathcal{I}_{t_1}^{\mathcal{C}_{\text{diag}}}$



$\mathcal{I}_{t_1}^{\mathcal{T}_{\text{diag}}}$



$\mathcal{I}_{t_1}^{\mathcal{K}_{\text{diag}}}$



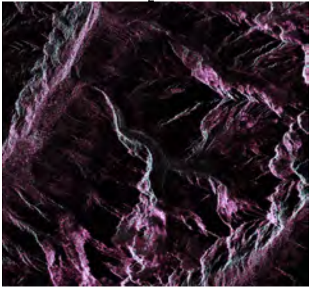
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

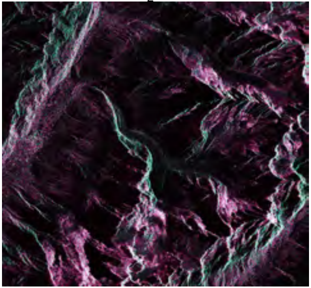
Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$t_2 = 2014 - 11 - 27$

$\mathcal{I}_{t_2}^{\mathcal{C}_{\mathtt{diag}}}$



$\mathcal{I}_{t_2}^{\mathcal{T}_{\mathtt{diag}}}$



$\mathcal{I}_{t_2}^{\mathcal{K}_{\mathtt{diag}}}$

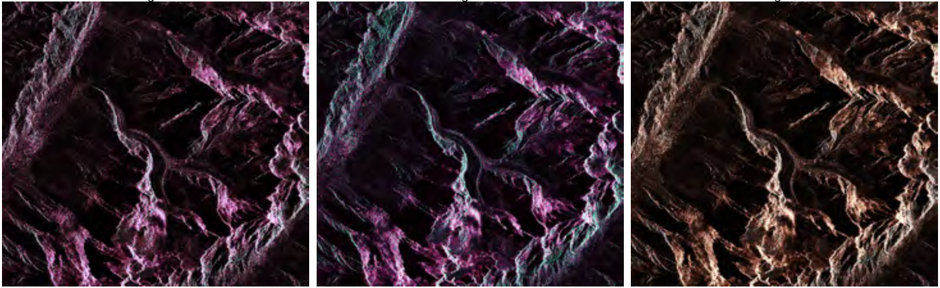
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$t_3 = 2014 - 12 - 09$

$\mathcal{I}_{t_3}^{\mathcal{C}\text{diag}}$ $\mathcal{I}_{t_3}^{\mathcal{T}\text{diag}}$ $\mathcal{I}_{t_3}^{\mathcal{K}\text{diag}}$



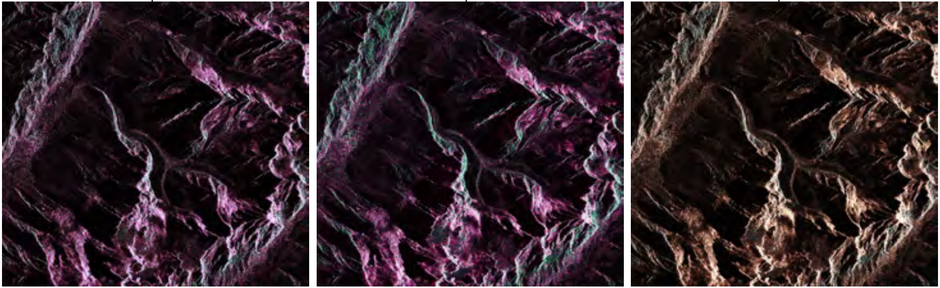
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / 5m × 20m Rg×Az

$t_4 = 2014 - 12 - 21$

$\mathcal{I}_{t_4}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_4}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_4}^{\mathcal{K}\mathtt{diag}}$



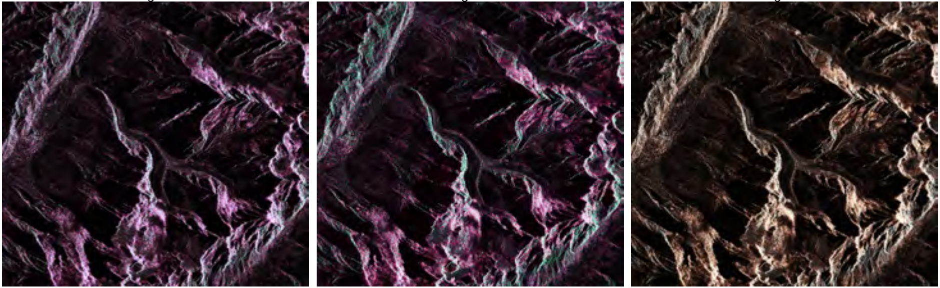
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / 5m × 20m Rg×Az

$t_5 = 2015 - 01 - 02$

$\mathcal{I}_{t_5}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_5}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_5}^{\mathcal{K}\mathtt{diag}}$

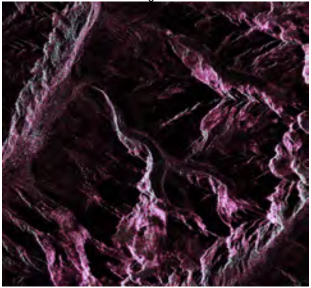
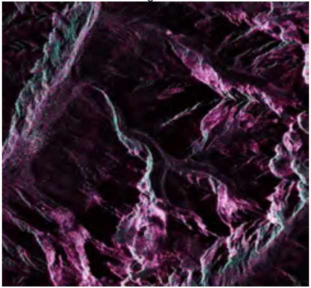
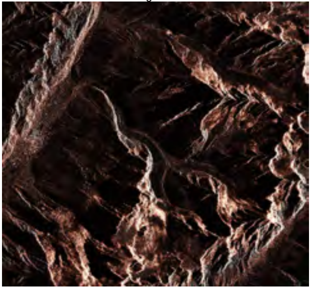


Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$$t_6 = 2015 - 01 - 14$$

$\mathcal{I}_{t_6}^{\mathcal{C}\text{diag}}$	$\mathcal{I}_{t_6}^{\mathcal{T}\text{diag}}$	$\mathcal{I}_{t_6}^{\mathcal{K}\text{diag}}$
		

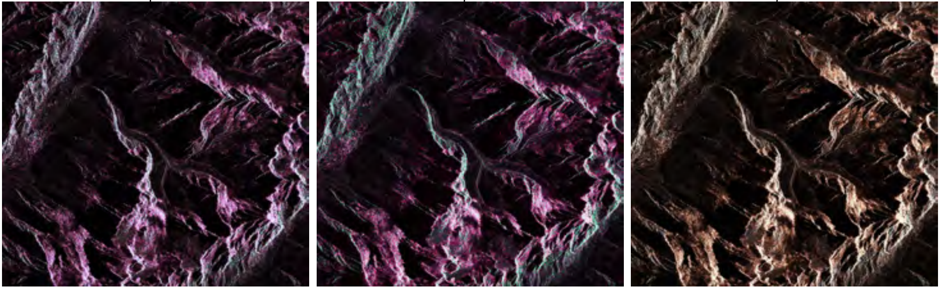
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$t_7 = 2015 - 01 - 26$

$\mathcal{I}_{t_7}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_7}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_7}^{\mathcal{K}\mathtt{diag}}$



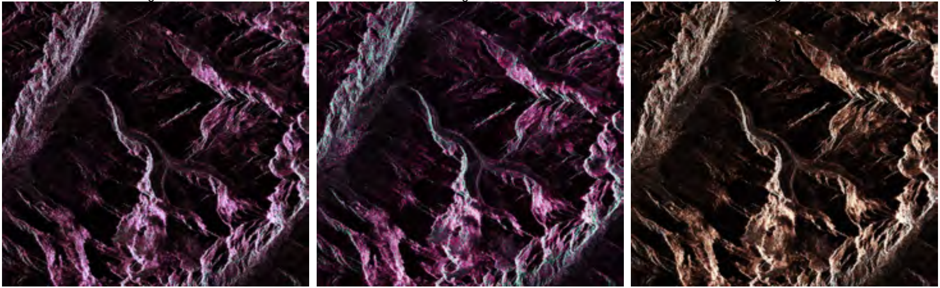
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$t_8 = 2015 - 02 - 07$

$\mathcal{I}_{t_8}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_8}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_8}^{\mathcal{K}\mathtt{diag}}$



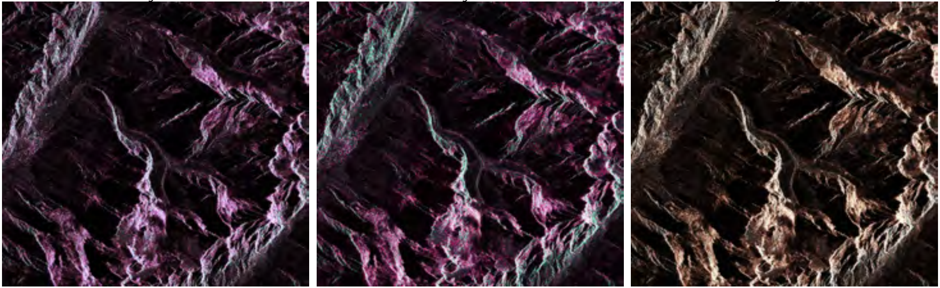
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / $5m \times 20m$ Rg×Az

$$t_9 = 2015 - 02 - 19$$

$\mathcal{I}_{t_9}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_9}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_9}^{\mathcal{K}\mathtt{diag}}$



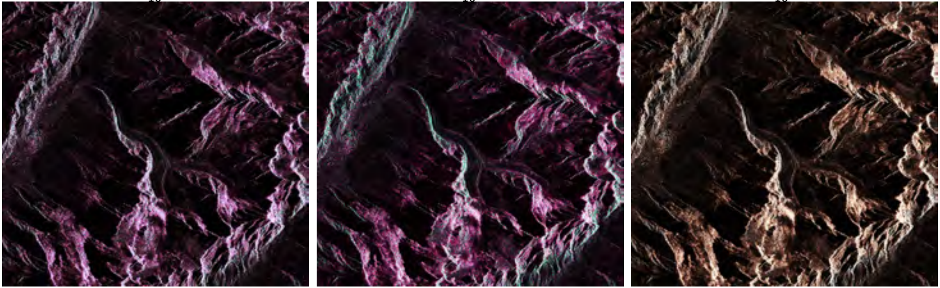
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / 5m × 20m Rg×Az

$t_{10} = 2015 - 03 - 03$

$\mathcal{I}_{t_{10}}^{\mathcal{C}\mathtt{diag}}$	$\mathcal{I}_{t_{10}}^{\mathcal{T}\mathtt{diag}}$	$\mathcal{I}_{t_{10}}^{\mathcal{K}\mathtt{diag}}$



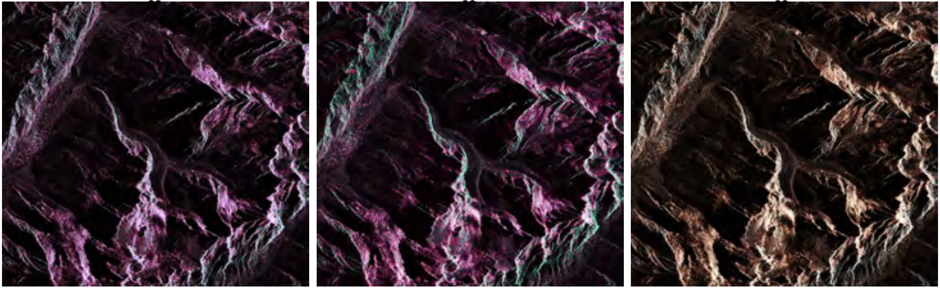
Sentinel-1A / Dual VV and VH PolSAR

Covariance, Coherency, Kennaugh

Interferometric wide swath mode / SLC / Descending / 43.1° incidence / C-Band / 5m × 20m Rg×Az

$t_{11} = 2015 - 03 - 15$

$\mathcal{I}_{t_{11}}^{\mathcal{C}\mathtt{diag}}$ $\mathcal{I}_{t_{11}}^{\mathcal{T}\mathtt{diag}}$ $\mathcal{I}_{t_{11}}^{\mathcal{K}\mathtt{diag}}$

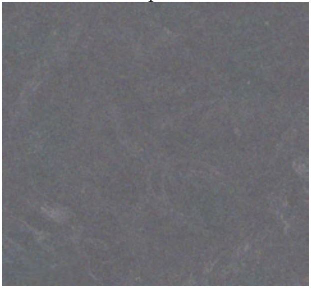
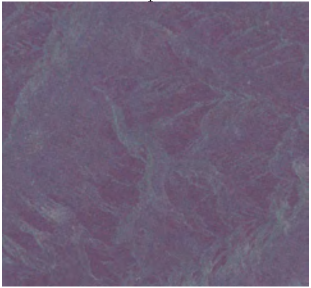
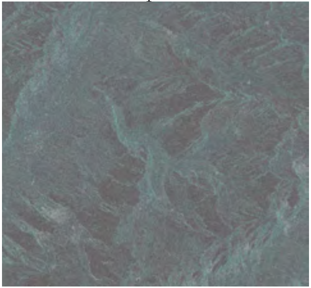


Sentinel-1A / Dual VV and VH PolSAR

Interferometric wide swath mode / SLC

Covariance, Coherency, Kennaugh

Wavelet Total Variation (WTV)

$\mathcal{I}_{t_1}^{\mathcal{C}\mathtt{diag}}$	$\mathcal{I}_{t_1}^{\mathcal{T}\mathtt{diag}}$	$\mathcal{I}_{t_1}^{\mathcal{K}\mathtt{diag}}$
		

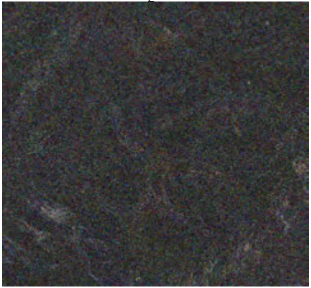
Sentinel-1A / Dual VV and VH PolSAR

Interferometric wide swath mode / SLC

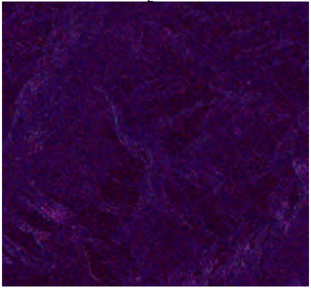
Covariance, Coherency, Kennaugh

WTV PolSAR sigmoid shrinkage

$\mathcal{I}_{t_2}^{\mathcal{C}\mathtt{diag}}$



$\mathcal{I}_{t_2}^{\mathcal{T}\mathtt{diag}}$



$\mathcal{I}_{t_2}^{\mathcal{K}\mathtt{diag}}$

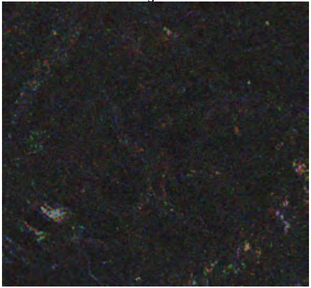
Sentinel-1A / Dual VV and VH PolSAR

Interferometric wide swath mode / SLC

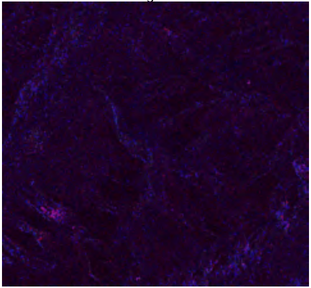
Covariance, Coherency, Kennaugh

WTV spatial recursive $\ell_1 - \ell_2$ regularization

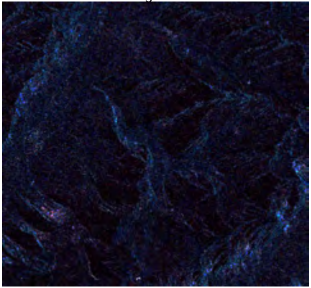
$\mathcal{I}_{t_3}^{\mathcal{C}_{\mathtt{diag}}}$



$\mathcal{I}_{t_3}^{\mathcal{T}_{\mathtt{diag}}}$



$\mathcal{I}_{t_3}^{\mathcal{K}_{\mathtt{diag}}}$



Conclusion

- High performance for Kennaugh polarimetric features in a context of wavelet analysis.
- Wavelet analysis at two different levels:
  - ⇒ Approximations / Temporal [Mean representatives of stable pixels/parts of the scene];
  - ⇒ Details / Spatio-Temporal [change-images representatives of the scene dynamics].
- Workable for long time series of high spatial resolution + multichannel,
  - ⇒ Wavelet on the temporal axis
  - ⇒ Shrinkage with respect to spatio-temporal change information
  - ⇒ Identifying stationary subsequences / seasonality.
- Easy monitoring of the temporal evolution of Alps glaciers.