Approches tomographiques structurelles pour l'analyse du milieu urbain par tomographie SAR THR

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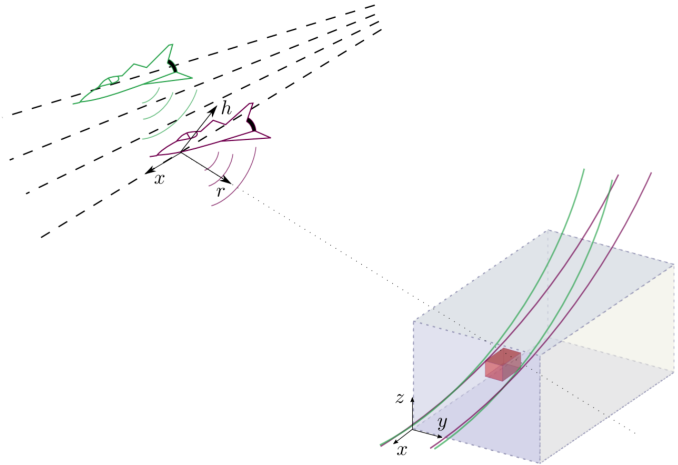
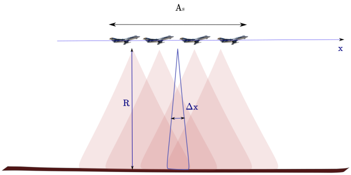
Principe

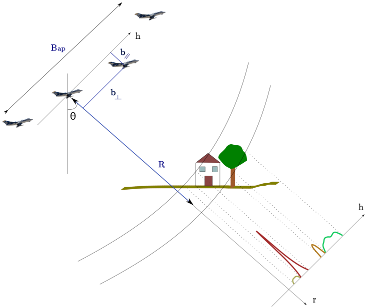
SAR tomography : Retrieve the $3D$ complex reflectivity from a pile of SAR SLC images in interferometric configuration



Principe

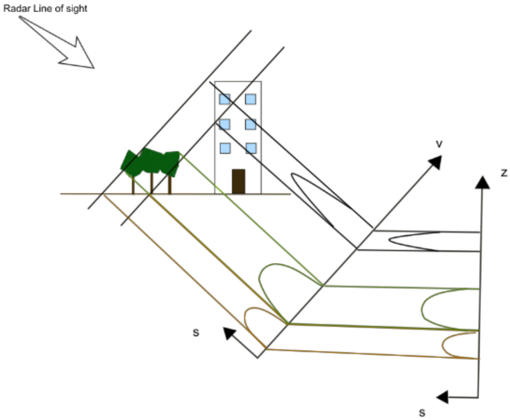
SAR Tomography is the natural extension of the $2D$: $3D$ SAR imaging is performed by using a synthetic aperture in the elevation direction





Tomographic inversion

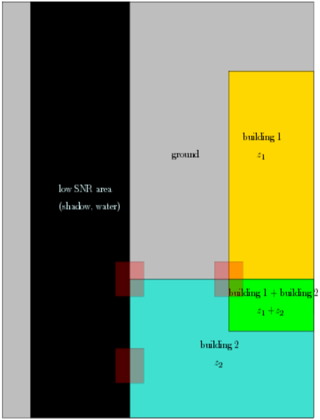
A lot of inversion techniques to retrieve the reflectivity : beamforming, Capon, maximum likelihood, MUSIC, M-RELAX...



A source is characterized by its reflectivity s and its location z

Similarity criterion

We need an estimation of the covariance matrix for the inversion. To do so we want to build a similarity map to do the averaging.



TerraSAR-X image of Paris

Similarity criterion : GLRT over Wishart distributed empirical matrix

We need a criterion to differentiate the pixels $\boldsymbol{y}$ that follow different distributions

Generalized Likelihood Ratio Test

Test on the empirical covariance matrix : $\hat{\boldsymbol{\Sigma}} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{y}_l \boldsymbol{y}_l^H$

$$\begin{cases} H_0 : & \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 \\ H_1 : & \boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2 \end{cases} \tag{1}$$

$$\mathrm{GLR}(\hat{\boldsymbol{\Sigma}}_1, \hat{\boldsymbol{\Sigma}}_2) = \frac{|\hat{\boldsymbol{\Sigma}}_1|^L |\hat{\boldsymbol{\Sigma}}_2|^L}{|\frac{1}{2}(\hat{\boldsymbol{\Sigma}}_1 + \hat{\boldsymbol{\Sigma}}_2)|^{2L}} \tag{2}$$

Similarity criterion : GLRT over Wishart distributed empirical matrix

Problem :

- With a large dimension M, we need to average a lot of samples to estimate the covariance matrix which destroys the structural information.
- When $\text{rank}\left(\hat{\boldsymbol{\Sigma}}_i\right) < M$ then, $\text{GLR}(\hat{\boldsymbol{\Sigma}}_1, \hat{\boldsymbol{\Sigma}}_2) = \frac{|\hat{\boldsymbol{\Sigma}}_1|^L |\hat{\boldsymbol{\Sigma}}_2|^L}{\frac{1}{2}|\hat{\boldsymbol{\Sigma}}_1 + \hat{\boldsymbol{\Sigma}}_2|^{2L}}$ is not defined
- The higher is the dimension, the more sensitive we are to the noise

Similarity criterion : GLRT over Wishart distributed empirical matrix

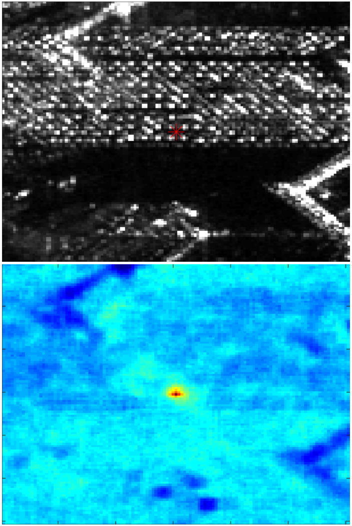
Solution ?
Rescale the non-diagonal elements of $\hat{\mathbf{\Sigma}}$:

$$\forall i \neq j, \hat{\mathbf{\Sigma}}_{ij} \leftarrow \gamma \hat{\mathbf{\Sigma}}_{ij} \text{ with } 0 \leq \gamma \leq 1 \quad (3)$$

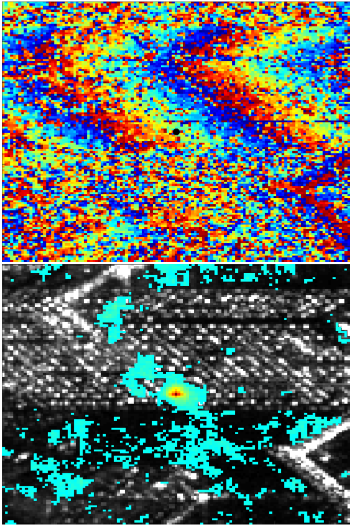
But...
By doing so we are less selective on the interferometric phase since :

$$\mathbf{\Sigma} = E\{\boldsymbol{y}\boldsymbol{y}^H\} = \begin{bmatrix} E\{|y_1|^2\} & \cdots & E\{y_1 y_M^*\} e^{\varphi_1 - \varphi_M} \\ \vdots & \ddots & \vdots \\ E\{y_M y_1^*\} e^{\varphi_M - \varphi_1} & \cdots & E\{|y_M|^2\} \end{bmatrix} \quad (4)$$

Similarity criterion : GLRT over Wishart distributed empirical matrix



GLRT on the whole window



Top 2000 points

Tomographic Model

$\boldsymbol{y}$ is the sum of the scatterers contribution in the resolution cell. We take into account permanent scatterers and speckle affected ones.

$$\boldsymbol{y} = \sum_{i=1}^{D_p} s_{p_i} \boldsymbol{a}(z_{p_i}) + \sum_{i=1}^{D_s} s_{s_i} \boldsymbol{a}(z_{s_i}) \odot \boldsymbol{x}_i + \boldsymbol{n} \quad (5)$$

where :

- $\boldsymbol{a}(z) = \left(e^{-ik_{z1}z} \cdots e^{-ik_{zM}z}\right)^T$
- s_{p_i} or s_{s_i} reflectivity of the respectively i*th* bright or decorrelated scatterer
- k_{z_j} wave number for the jth antenna
- $\boldsymbol{x}_i$ multiplicative noise related to the ith source. $\boldsymbol{x}_i$ follows a complex circular Gaussian with zero mean and unknown covariance matrix $\boldsymbol{C}_i$.
- $\boldsymbol{n}$ additive white noise
- $\odot$ elementwise multiplication (Schur-Hadamard product)

Tomographic Model

$\boldsymbol{y}$ follows a complex circular Gaussian with non-centered mean and covariance matrix $\boldsymbol{\Sigma}$:

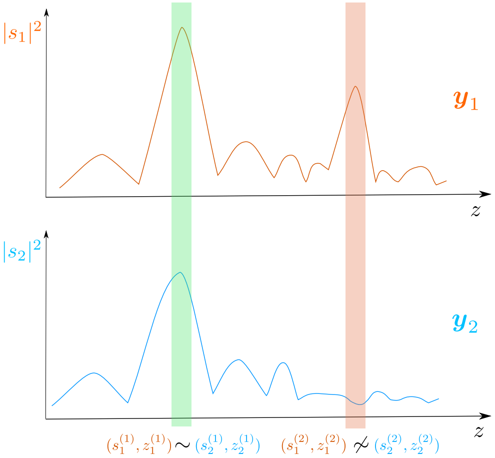
$$\mathrm{p}(\boldsymbol{y}|\boldsymbol{\Sigma}, \boldsymbol{s}, \boldsymbol{z}) = \frac{1}{\pi^n |\boldsymbol{\Sigma}|} \exp\left[-\bar{\boldsymbol{y}}^H \boldsymbol{\Sigma}^{-1} \bar{\boldsymbol{y}}\right] \quad (6)$$

where :

- $\bar{\boldsymbol{y}} = \boldsymbol{y} - \sum_{i=1}^{D_p} s_{p_i} \boldsymbol{a}(z_{p_i})$
- $\boldsymbol{\Sigma} = \sum_{i=1}^{D_s} |s_i|^2 \boldsymbol{C}_i \odot \left(\boldsymbol{a}(z_i)\boldsymbol{a}^H(z_i)\right) + \sigma_n^2 \boldsymbol{I}_M$

Similarity Criterion

To compare $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$, we extract the main stables contributions and test if they are likely to explain both pixels



Similarity Criterion

For each pair of scatterer :

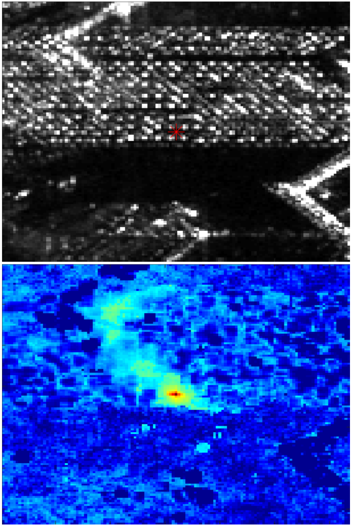
$$\begin{cases} H_0^{(k)} : & s_1^{(k)} = s_2^{(k)} = s_{12}^{(k)} \quad \text{and} \quad z_1^{(k)} = z_2^{(k)} = z_{12}^{(k)} \\ H_1^{(k)} : & s_1^{(k)} \neq s_2^{(k)} \quad \text{or} \quad z_1^{(k)} \neq z_2^{(k)} \end{cases} \tag{7}$$

$$\mathcal{L}_G^{(k)} = \frac{\mathrm{p}(\boldsymbol{y}_1, \boldsymbol{y}_2 | H_0, \hat{\boldsymbol{\Sigma}}_{12}^{(k)}, \hat{s}_{12}^{(k)}, \hat{z}_{12}^{(k)})}{\mathrm{p}(\boldsymbol{y}_1, \boldsymbol{y}_2 | H_1, \hat{\boldsymbol{\Sigma}}_1^{(k)}, \hat{s}_1^{(k)}, \hat{z}_1^{(k)}, \hat{\boldsymbol{\Sigma}}_2^{(k)}, \hat{s}_2^{(k)}, \hat{z}_2^{(k)})} \tag{8}$$

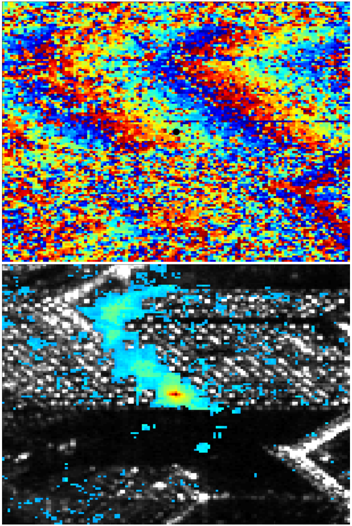
Then the final criterion is :

$$\mathcal{L} = \sum_k \log \mathcal{L}_G^{(k)} \tag{9}$$

Similarity criterion proposed

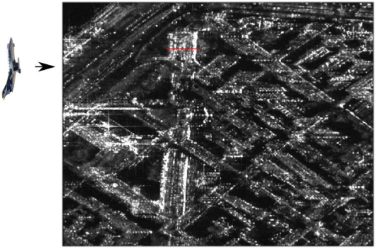


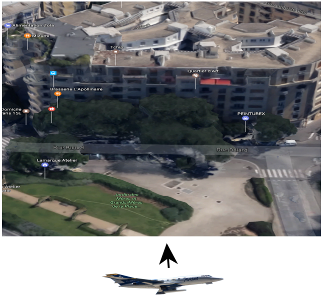
Similarity over the window



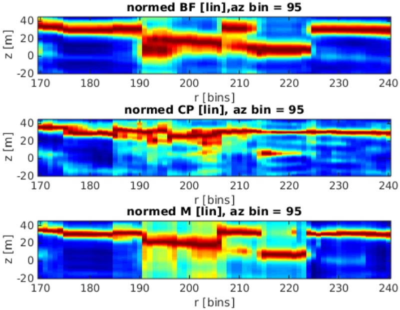
Top 2000 points

Tomographic reconstruction

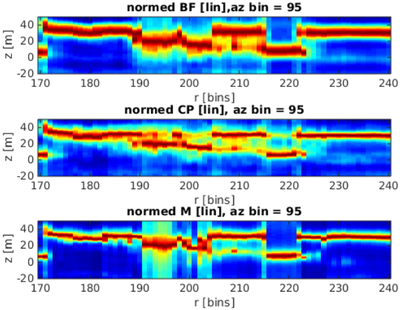




Tomographic reconstruction

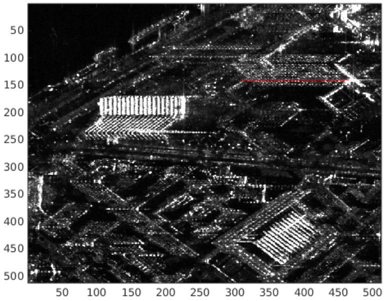


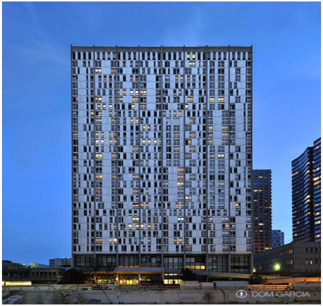
Estimation via gausscar



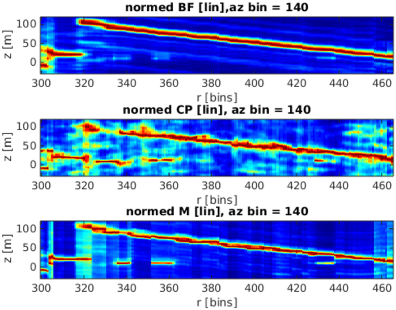
Proposed method

Tomographic reconstruction

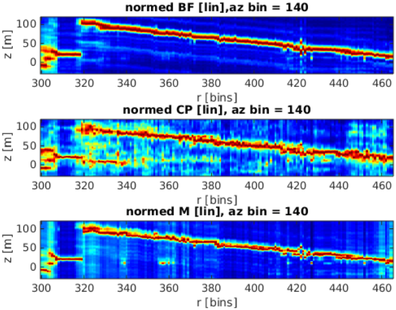




Tomographic reconstruction

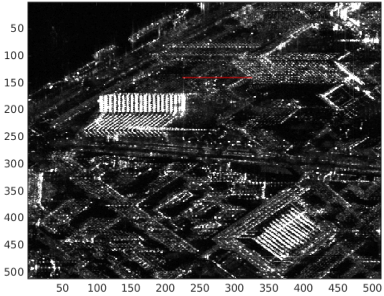


Estimation via gausscar

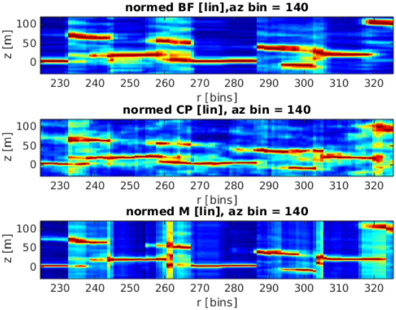


Proposed method

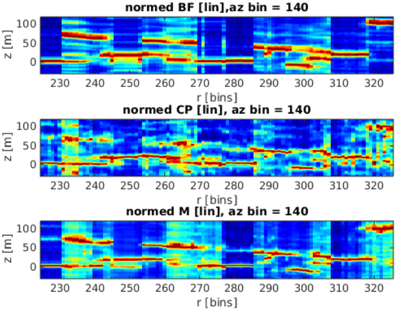
Tomographic reconstruction



Tomographic reconstruction



Estimation via gausscar



Proposed method

Conclusion and perspectives

Conclusions

- Better selection of the samples for the estimation of the covariance matrix
- Improvement in the sources separation

Perspectives

- Try to keep only the best samples with rank deficient estimators
- Optimize the combination of the tests
- Buid a strategy based on the equivalent number of looks to optimize the sample selection